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I.Sc-Class-XII, Relations and Functions Ex.1.1. Date-09-5-2020

Q.1. Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x,y) : x$ and y have same number of pages$\}$, is an equivalence relation.

Soln: Here, we see that set A is the set of all books in the library of a college. and $R = \{(x,y) : x$ and y have the same number of pages$\}$.

Now, R is reflexive, since $(x,x) \in R$ and x has the same number of pages.

Let $(x,y) \in R$

$\Rightarrow$ x and y have the same number of pages.

$\Rightarrow$ y and x have the same number of pages.

$\Rightarrow$ $(y,x) \in R$. So, R is symmetric.

Now, let $(x,y) \in R$ and $(y,z) \in R$

$\Rightarrow$ x and y have the same number of pages and y and z have the same number of pages.

$\Rightarrow$ x and z have the same number of pages

$\Rightarrow$ $(x,z) \in R$.

$\therefore$ R is transitive. Hence, R is an equivalence relation.

Q.2. Show that the relation R in the set $A = \{1,2,3,4,5\}$ is given by $R = \{(a,b) : |a-b|$ is even$\}$, is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other, but no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

Soln It is given that $A = \{1,2,3,4,5\}$, $R = \{(a,b) : |a-b|$ is even$\}$

Let $a \in A \Rightarrow |a-a| = 0$ (which is even), $\forall a$.

So, R is reflexive.

Let $(a,b) \in R \Rightarrow |a-b|$ is even $\Rightarrow |-(b-a)| = |b-a|$ is also even.

$\Rightarrow (b,a) \in R$. So R is symmetric.

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Rest part of Q.9.

Now, Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even.

$\Rightarrow (a-b)$ is even and $(b-c)$ is even.

$\Rightarrow (a-c) = (a-b)+(b-c)$ is even.

$\Rightarrow |a-c|$ is even $\Rightarrow (a,c) \in R$.

So, R is transitive. Hence, R is an equivalence relation.

Now, all the elements of the set $\{1,3,5\}$ are related to each other as all the elements of this set are odd. So, the modulus of the difference between any two elements will be even.

Similarly, all the elements of the set $\{2,4\}$ are related to each other as all the elements of this set are even. So, the modulus of the difference between any two elements will be even.

Also, no element of the set $\{1,3,5\}$ can be related to any element of $\{2,4\}$ as all elements of $\{1,3,5\}$ are odd and all elements of $\{2,4\}$ are even. So, the modulus of the modulus of the difference between the two elements will not be even.

Q. 3. Define identity relation.

Ans: Identity relation: $\rightarrow$ Any relation on a set A is called identity relation if

$$R = \{(a,b) : a \in A,\ b \in A \text{ and } a=b\}$$

In this way identity relation $R = \{(a,a) : \forall a \in A\}$

It is denoted by I_A. For example: Let $A = \{1,2,3,4\}$.
then, $I_A = \{(1,1), (2,2), (3,3), (4,4)\}$